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IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2022

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

DESE50002 – Electronics 2

Date: 3 May 2022 10.00 to 11.30 (one hour thirty minutes)

This paper contains 6 questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate how the examiners intend to distribute the marks for this paper.

This is an OPEN BOOK Examination.

1. a) For the signal $x(t)$ shown in *Figure Q1a*, and given that $u(t)$ is the unit step function, sketch each of the following signals on your answer sheet:

   (i) $x(t)u(t)$

   (ii) $x(t-4)$

   (iii) $x(2t-4)$

   (iv) $x(2-t)$

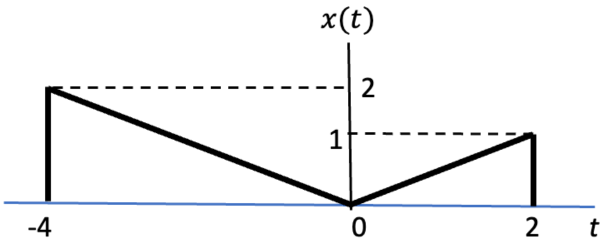

Figure Q1a

[8]

b) *Figure Q1b* shows a continuous-time signal $y(t)$. If $y(t)$ is sampled at 1Hz, write down the mathematical model for the sampled discrete-time signal $y[n]$ in terms of the unit impulse function $\delta(t)$.

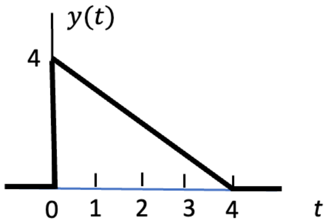

Figure Q1b

[4]

c) A continuous-time signal $s(t) = 2\cos(2764.6t - \pi/2)$ is sampled at 3,520Hz. Sketch on your answer sheet the discrete-time sampled signal $s[n]$ for one complete cycle of $s(t)$.

[6]

d) *Figure Q1d* shows two discrete-time signals $x_1[n]$ and $x_2[n]$. Sketch on your answer sheet the signals $y_1[n]$ and $y_2[n]$ where:

   (i) $y_1[n] = x_1[n] + 2x_2[n]$

   (ii) $y_2[n] = x_1[n]\,x_2[n]$

[4]

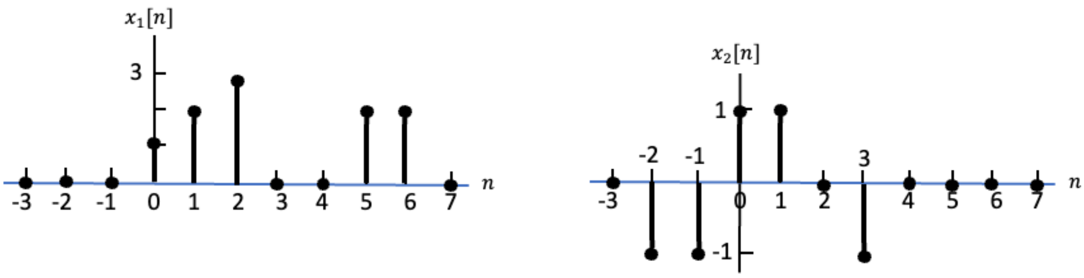

Figure Q1d

2. a) The rectangular function $\mathrm{rect}(t)$ is defined as:

$$\mathrm{rect}(t) = \begin{cases} 0, & \text{if } |t| > \frac{1}{2} \\ \frac{1}{2}, & \text{if } |t| = \frac{1}{2} \\ 1, & \text{if } |t| < \frac{1}{2} \end{cases},$$

On your answer sheet, sketch the signals $\mathrm{rect}\left(\frac{t}{2}\right)$ and $\mathrm{rect}\left(\frac{t}{2\tau}\right)$.

[4]

b) The Fourier transform $X(\omega)$ of a signal $x(t)$, is defines as:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Proof from first principles that if $x(t) = \mathrm{rect}\left(\frac{t}{2\tau}\right)$, then $X(\omega) = 2\tau\ \mathrm{sinc}(\omega\tau)$

[10]

3. The call signal $x(t)$ of an owl falls within the frequency range of 550Hz to 18kHz. *Figure Q3a* shows the frequency spectrum $X(\omega)$ of such an owl call.

a) Sketch on your answer sheet the frequency spectrum $X_s(\omega)$ of the sampled signal $x[n]$ given that the sampling frequency is 44.1kHz. Your sketch should have a frequency scale between ±100kHz.

[8]

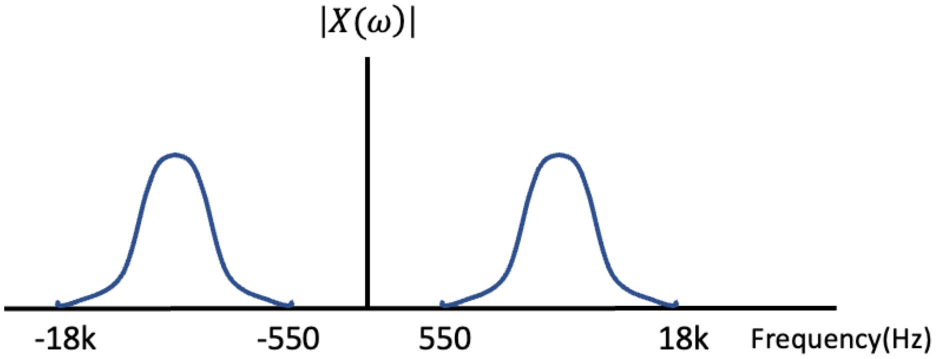

Figure 3a

b) It is known that the area is also inhabited by bats. They perform echolocation using single tone sinusoidal signal at a frequency above 18kHz. A recording of the owl call shows a strong frequency component at 15kHz in addition to the normal owl call spectrum. Given that this spurious frequency component is caused by the bats because of aliasing effect, derive with justifications the bat's echolocation signal frequency.

[6]

4. *Figure Q4* shows a continuous-time system consisting of two integrators, three scalar multipliers and an adder.

a) Show that the differential equation that relates the output $y(t)$ to the input $x(t)$ is given by:

$$\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$

[10]

b) Using a), or otherwise, derive the transfer function $H(s) = Y(s)/X(s)$.

[2]

c) Given that $b_0 = 100,\ a_1 = 20$ and $a_0 = 100$, derive the DC gain, natural frequency of the damping factor of the system.

[5]

d) What is the gain of the system at frequency $\omega = 1$ and 10 rad/sec.

[3]

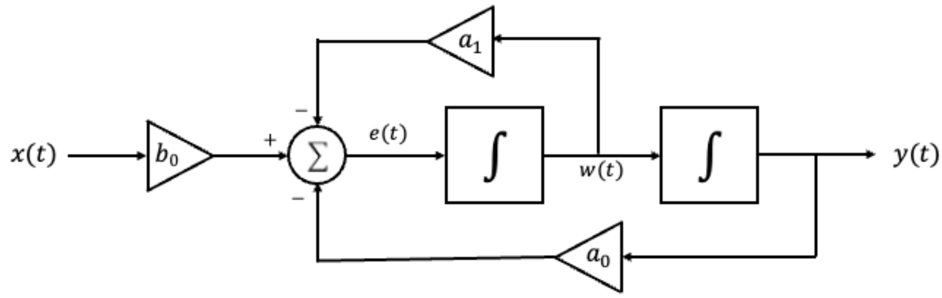

Figure Q4

5. A discrete-time system has an impulse response $h[n]$ given by:

$$h[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3].$$

a) Plot the impulse response $h[n]$ of the system.

[4]

b) What is the transfer function H[z] of the system?

[4]

c) A causal signal $x[n]$ shown in Figure Q5 is applied to the input of the system. Derive the output $y[n]$ for $0 \leq n \leq 6$.

[8]

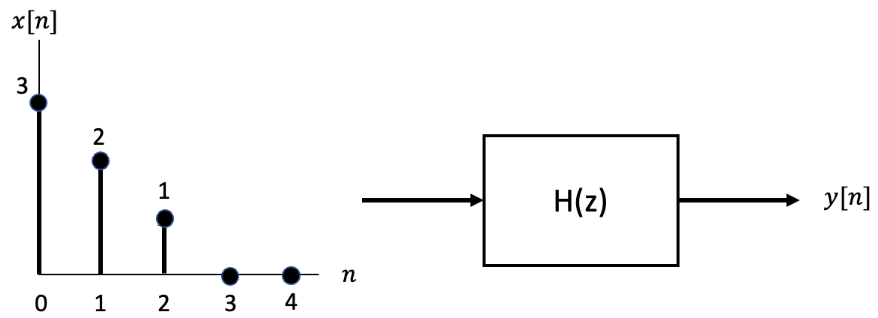

Figure Q5

6. *Figure Q6* show a simple proportional feedback system to control the motor speed y(t) in response to the set-point x(t) in the s-domain. The transfer function of the motor is $G(s) = \frac{10}{0.1s+1}$. The proportional controller has a gain of K_p.

a) What is the DC gain and the time constant of the motor?

[2]

b) Derive the closed-loop transfer function of the system Y(s)/X(s).

[4]

c) Given that $x(t) = 5u(t)$ and $K_p = 20$, calculate the steady-state error $e(t)$ and the time constant of the closed-loop system.

[8]

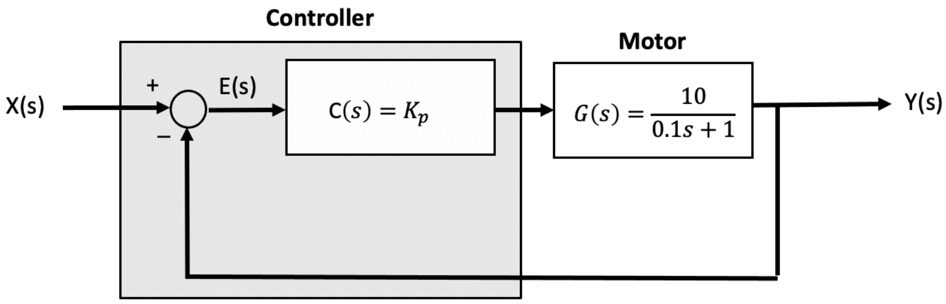

Figure Q6

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